

Relations

If things with properties exist in space-time, and some things or properties imply others, then one of the modes of existence which needs to be explained (for a good ontological theory) is the general idea of a 'relation' between things. At the very least there seem to be spatial, temporal, causal, and logical relations, and if we are thinking about the structure of reality rather than its contents, then relations will be the most significant part of reality. Relations in ordinary life can be fairly imprecise, both in their meaning and in their application (as in the case of resemblance, or spatial proximity), but the formal aspects of relations can be specified quite precisely.

In logic we express a relation R between two objects a and b as ' aRb ' or ' Rab ' (where the order of the small letters matters, because relations often have a direction). Rab is a 'diadic' relation, because two objects are involved; 'between' is a triadic relation ($Rabc$), and higher levels are possible. It is acceptable to write ' Raa ', which means that a bears the relation R to itself ('his own worst enemy'). Using these symbols we can identify various types of relation, and even apply formal logic to them. Thus a relation between a thing and itself is called a '**reflexive**' relation (Raa). A two-way relation (like marriage) is called '**symmetric**' (because Rab and Rba imply one another); being a father is 'asymmetric', because Rab makes Rba impossible. A 'non-symmetric' relation neither requires symmetry nor rules it out. Relations which carry over from one relation to the next (such as identity) are called '**transitive**'; for example, if one billiard ball looks identical to a second, and the second to a third, then the first and third must look identical (if Rab and Rbc , then Rac). These three (reflexivity, symmetry and transitivity) are the main features of relations, and they can occur alone or in combinations. We also categorise some relations as being 'one-many' (being the chief of a tribe), or 'many-one' (being in a queue).

A particularly significant relation is where R is reflexive and symmetric and transitive, and is called an '**equivalence**' relation. If R applies to some members of a group of entities, it 'partitions' them off from the rest, since those will all bear the relation R to one another, and the others will be excluded; an example would be pupils in a school who are the same height. Another interesting case is the '**ancestral**' relation, where a chain formed by a relation implies a new generalised relation represented by the chain as a whole, as in the case of the chain produced by 'parent of', which implies the further general relation 'ancestor of'.

An interesting question is whether relations should be understood as internal or external to the participants in the relation (the 'relata'). The natural thought is that two objects stand alone, with internal features, and then a relation between them adds something extra, which must be external to the objects. However, this seems to give the relation an existence of its own, making it a third party in the situation. This invites the question of how this extra entity – the relation – relates to the two objects involved. It seems that two further relations are required, to connect the relation to the two relata – but we immediately see an infinite regress of relations appearing, so this would be ridiculous. The conclusion is that external relations must be impossible, so all relations are internal to the relata involved.

An example of an internal relation might be the succession of natural numbers, where being a successor is part of the nature of each number. However, the general idea that your being to my left is part of my nature seems speculative and rather odd, implying the large metaphysical claim that reality is highly unified by each thing having aspects that mirror all the other things, rather than containing separated items. It may be better to embrace externalism, and add separate relations to our ontology, perhaps as universals. A widely held alternative, though, is to say that relations do not actually exist. If I stand in one place and you in another, it is not clear that your being to my left is some further fact. If I push and a door opens, there have long been doubts about whether some further relation called 'causation' must be invoked to explain it. If one thing being true makes another connected thing true, do we need a distinct relation called 'implication' to explain the link?

Before we abandon relations, reducing them to other aspects of reality, we must face up to the fact that relations have many properties which remain the case even when the relata are ignored, or don't even exist. In arithmetic we accept the numbers, with all of their features, but we must also accept the operators, such as '=' or '+', which are relational, and have their own rules. If I stand somewhere, there seems to be a relation of 'to my left' even when no one is occupying that location. I can instruct people to form a straight line, so they understand the relation before it exists.

An interesting phenomenon is that a road has a direction because of where it is laid, but it also has a direction to the town and from the town, which we impose on the facts about the road (implying that relations are partly objective and partly conventional). In writing ' Rab ' we attend to the ordering of a and b , because this often matters, but this suggests two different sorts of relation, where order does or does not matter. If a is 'above' b , the order is built in, but this immediately implies a mirror relation of 'below'. We talk of you being to my left and my being to your right, which seem to be two aspects of a single relation, implying the existence of something objective which can be described in a variety of ways. The relation of 'near', on the other hand seems to be independent of this directionality, since saying I am near you or you are near me convey the same information. The two types may just be paraphrases of one another, or they may indicate two different families of relations.

The problem with entirely denying the existence of relations is that some aspects of reality, such as time and space, seem to be intrinsically relational, so that you cannot even talk about them without believing in relations like before/after or near/far. One might go further, and say it is not just 'some aspects' of reality that are relational, but that reality is entirely relational in character, and our perception of it is intrinsically relational in every way. If you look out of the window, what you seem to experience is patterns and structures, before you take in the details that constitute them. Many mathematicians take their subject to be the study of patterns, and all of the physical sciences can be presented as a study of the structure of reality, rather than of the particular ingredients. This has the intriguing implication that relations might be more important than the objects, and we could consider eliminating the objects, explaining them all in relational terms.